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The Use of Hurst and Effective Return in Investing

by

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THE USE OF HURST AND EFFECTIVE RETURN IN INVESTING

Abstract

We present a look at the pathwise properties of mutual funds via the Hurst exponent, as well as ways to evaluate performance via Effective Return. Both methodologies are examined in the context of distributional properties and tail analysis, as well as the linear and nonlinear dependence of the volatility of returns in time.

Empirical tests comparing the use of Hurst and Effective Return against more traditional measures such as the Sharpe Ratio and Mean-Variance Optimization are done. These tests indicate that both Hurst and Effective Return are more robust to the clustering of losses than traditional measures and have the ability to fully characterize the behavior of mutual funds.

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Introduction

Empirical finance focuses on the study, from a statistical point of view, of the behavior of data obtained from financial markets in the form of time series. This paper will provide a brief introduction to the field, which will supply the necessary background to introduce two new measures that can be used to make more effective investment decisions.

Since the statistical study of market prices has gone on for more than half a century, one might wonder whether there is anything new to say about the topic today. The answer is definitely *yes,* for various reasons.

The first is that, with the advent of electronic trading systems and the computerization of market transactions, quotes and transactions are now systematically recorded in major financial markets all over the world, resulting in a database size surpassing any econometricians may have dreamed of in the 1970s.

Moreover, whereas previous data sets were weekly or daily reports of prices and trading volumes, these new data sets record all transactions and contain details of intraday tick-by-tick price dynamics, thus providing a wealth of information that can be used to study the role of market micro-structure in price dynamics. At the same time, these high-frequency data sets have complicated seasonalities and new statistical features, the modeling of which has stimulated new methods in time series analysis.

Last, but not least, the availability of cheap computing power has enabled researchers and practitioners to apply various nonparametric methods based on numerical techniques for analyzing financial time series. These methods constitute a conceptual advance in the understanding of the properties of these time series, since they make very few *adhoc* hypotheses about the data and reveal some important qualitative properties on which models can then be based.

The Hurst exponent and Effective Return are examples of these nonparametric techniques that are uniquely qualified to "let the data speak for itself." And while, like other nonparametric techniques, Hurst and Effective Return provide qualitative information about financial time series, they can be converted into semiparametric techniques that can, without completely specifying the form of the price process, imply the existence of a parameter that *does* describe the process.

In Section 2 of this paper we will cover the statistical analysis of asset price variation, specifically mutual fund price variation. In Section 3 equity curves and Effective Return will be discussed. In Section 4 tests will be conducted that show the robustness of the measure(s) versus traditional measures. Section 5 is the conclusion.

2. Statistical Analysis of Asset Price Variations

In this section we will describe some of the statistical properties of log return series obtained from mutual funds. The properties of these time series can be separated into two types: marginal properties and dependence properties. Section 2.1 focuses on the marginal distribution of mutual fund returns. Section 2.2 discusses the dependence properties of mutual funds across time.

2.1 Distributional Properties of Mutual Fund Returns

Empirical research in financial econometrics in the 1970s concentrated mostly on modeling the unconditional distribution of returns, defined as:

$$F_T(u) = P(r(t,T) \leq u)$$

where $r(t,T)$ are the log returns for the financial asset at time t over time horizon T. One can summarize the empirical results by saying the distribution F_T tends to be non-Gaussian, sharp peaked, and heavy tailed, with these properties being more pronounced for intraday values of T ($T \leq 1$ day). The methods described below attempt to measure these properties and quantify them in a precise way.

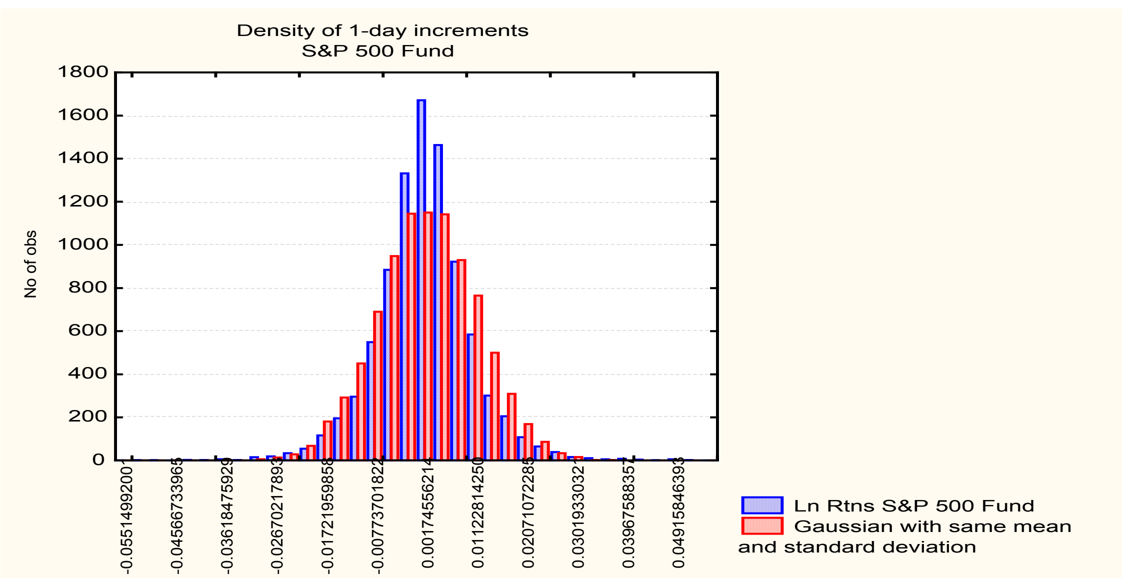
2.1.2 Marginal distribution features

As early as the 1960s Mandelbrot [1] pointed out the insufficiency of the normal distribution for modeling the marginal distribution of asset returns and their heavy-tailed character. Since then the non-Gaussian character of the distribution of price changes has been repeatedly observed in various market data. One way to quantify the deviation from the normal distribution is by determining the kurtosis of the distribution F_T, defined as:

$$\kappa = [E[(r(t,T) - E(r(t,T))^4)]>/\sigma(T)^2] - 3$$

where $\sigma(T)^2$ is the variance of the log returns $r(t,T) = x(t+T) - x(t)$. The kurtosis is defined such that $\kappa = 0$ for a Gaussian distribution, with a positive value of κ indicating a "fat tail," that is, a slow asymptotic decay of the probability distribution function (PDF). The kurtosis of the increments of mutual funds are far from Gaussian values: typical values for $T = 1$ day are: $\kappa \approx 7$ for an S&P 500 fund and $\kappa \approx 44$ for an emerging markets fund.

Figure 1



The non-Gaussian character of the distribution makes it necessary to use other measures of dispersion than standard deviation to capture the variability of returns. More generally, one can consider higher-order moments or cumulants as measures of dispersion/variability. The kth central moment of the absolute returns is defined as:

$$\mu_k(T) = E\,|r(t,T) - Er(t,T)|^k$$

Under the hypothesis of stationary returns, $\mu_k(T)$ should not be dependent on t. However, it is not obvious *a priori* whether the moments are well-defined quantities: their existence depends on the tail behavior of the distribution F_T. This leads to another measure of variability, the tail index of the distribution of returns (defined as the order of the highest finite absolute moment):

$$\alpha(T) = sup\{k>0,\ \mu_k(T)<+\infty\}$$

Note that $\alpha(T)$ depends *a priori* on the time resolution T. One can define in an analogous way a left tail index and a right tail index by taking one-sided moments. There are many estimators for $\alpha(T)$, the best known being the Hill and the Pickands estimators [2]. In the case of daily returns for mutual funds, one obtains an estimator between 1.5 and 4 [paper by Clark, forthcoming], indicating heavy tails of regularly varying type. The implications of these values are that most funds on a daily basis have a finite variance, but the kurtosis of the fourth moment is apparently infinite/unbounded.

However the Hill estimator and Pickands estimators are very sensitive to dependence in the data [2] as well as to sample size. It has been our experience, as well as others [2], that tail estimation methods do not allow for a precise conclusion concerning estimates of $\alpha(T)$ either at individual or across different time scales, so these results must be interpreted with caution.

2.1.3 Scaling Properties

Most econometric studies of financial time series in the 1980s dealt with asset returns on a single time scale, typically daily. Although daily returns are frequently used by the market to measure investment turnover, investors and market operators are also concerned with price variations on other time horizons, ranging from intraday time scales, e.g., several minutes for traders to several weeks or months for fund managers. An interesting and relevant question to study then is how F_T changes with T. More specifically, one would like to identify statistical quantities that are scale-invariant, i.e. have the same value across different time resolutions, which would lead to interesting links between the statistical behavior of the returns at different time scales.

While the variance of F_T is observed to be approximately linear as a function of T - for at least 80% of mutual funds – higher cumulants and moments exhibit behaviors deviating from the random walk model. These deviations are linked to the dependence structure of the returns (see Section 2.2). In the case of the 20% of mutual funds that do not scale linearly, this is because the underlying PDF of $r(t,T)$ apparently follows a pure Levy process - the variance is infinite if the characteristic index α is between [1,2) while a Levy process with an α less than 1 has an infinite mean and an infinite variance though the median still exists. And these 20% of funds do indeed have an α less than 1.

2.2 Dependence properties

2.2.1 Absence of linear correlations

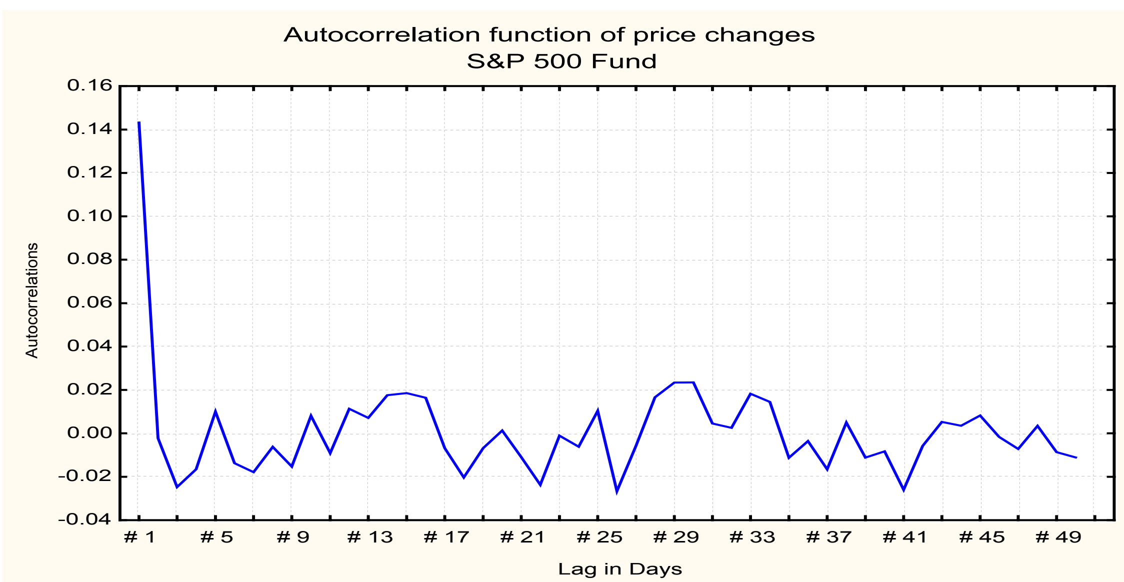
It is a well known fact that price movements in liquid markets do not exhibit any significant autocorrelations; the autocorrelation function of the price changes:

$$C(\tau) = [E(r(t,T)r(t+\tau,T)) - E(r(t,T)E(r(t+\tau,T)]/var[r(t,T)]$$

rapidly decaying to zero in a few minutes. For $T \geq 15$ minutes it can be safely assumed to be zero for all practical purposes [3]. The absence of significant linear correlations in price increments has been widely documented [4, 5] and is often cited as support for the Efficient Market Hypothesis (EMH). The absence of correlation is easy to understand: if the price changes exhibit significant correlation, this correlation may be used to conceive a simple strategy with positive expected earnings. Such strategies, termed arbitrage, will therefore tend to reduce correlations except for very short time intervals, which represent the time the market takes to react to new information. This correlation time for organized futures markets is typically several minutes and for foreign exchange markets even shorter.

The fast decay of the autocorrelation function implies the additivity of variances; for uncorrelated variables the variance of the sum is the sum of the variances. The absence of linear correlation is thus consistent with the observed linear increases of variance with respect to time scale.

Figure 2



2.2.2 Volatility Clustering

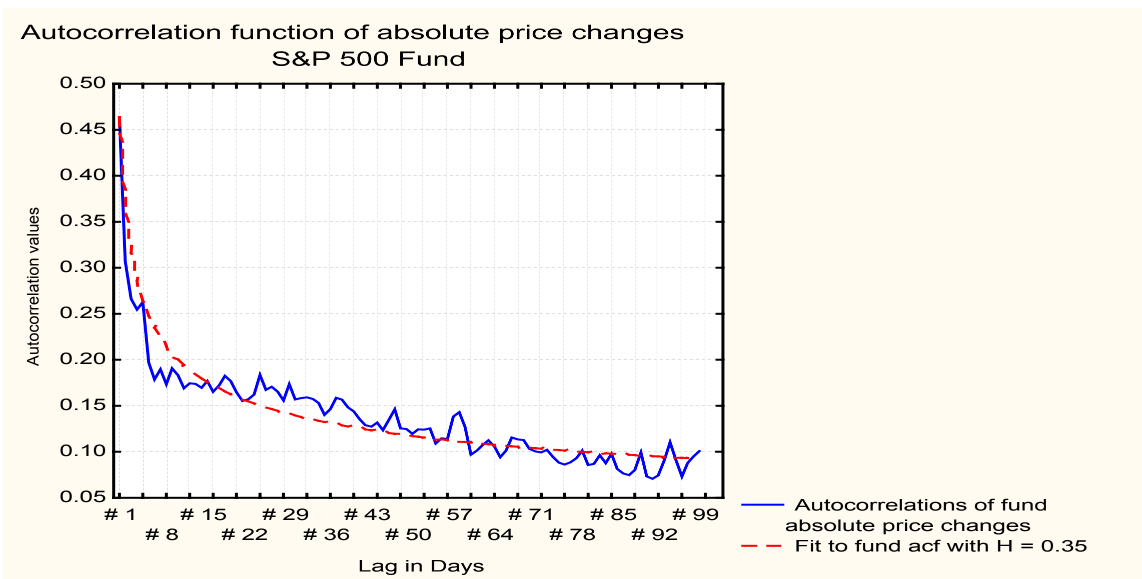
However, the absence of serial correlations does not imply the independence of the increments; for example, the square of the absolute value of price changes exhibits slowly decaying serial correlations. This can be measured by the autocorrelation function $g(\tau)$ of the absolute value of the increments, defined as:

$$g(\tau) = [E(abs(r(t,T))abs(r(t+\tau,T))) - E(abs(r(t,T))E(abs(r(t+\tau,T))]/var[abs(r(t,T))]$$

For the S&P 500 fund examined before, the slow decay of $g(\tau)$ is well represented by a power law [6]:

$$g(\tau) = g_0/\tau^H \quad H = 0.35 \pm 0.034$$

Figure 3



This slow relaxation of the correlation function *g* of the absolute value of returns indicates persistence in the *scale* of fluctuations. This phenomenon can be related to the clustering of volatility, well known in the financial literature, where a large price movement tends to be followed by another large price movement but not necessarily in the same direction.

2.3 Pathwise Properties

One of the main issues in financial econometrics is to quantify the notion of *risk* associated with a financial asset or portfolio of assets. The risky character of a financial asset is associated with the irregularity of the variations of its market price: risk is therefore related to the (un)smoothness of the trajectory. This is one crucial aspect of the empirical data one would like to have a mathematical model to reproduce.

Each class of stochastic models generates sample paths with certain local regularity properties. In order for the model to adequately represent the intermittent character of price variations, the local regularity of the sample paths should try to reproduce those empirically observed price trajectories.

2.3.1 Holder Regularity

In mathematical terms the regularity of any function may be characterized by its local Holder exponents. A function f is h-Holder continuous at point t_0 if and only if there exists a polynomial of degree < h such that

$$|f(t) - P(t - t_0)| \leq K_t \, | t - t_0 |^h$$

Let $C^h(t_0)$ be the space of real-valued functions that verify the above property at t_0. A function f is said to have a local Holder exponent α if for $h<\alpha, f \in C^h(t_0)$ and $h>\alpha, f \notin C^h(t_0)$. Let $h_f(t)$ denote the local Holder exponent of f at point t.

In the case of sample path $X_t(\varpi)$ of a stochastic process, X_t, $h_{X(\varpi)}(t) = h_\varpi(t)$ depends on the particular sample path considered, i.e., on ϖ. However, there are some famous exceptions: for example, Fractional Brownian Motion (FBM) with the self-similarity parameter $h_B = 1/H$ with probability 1, i.e., for almost all sample paths. Note that no such results hold for sample paths of Levy processes or even stable Levy processes.

Given that the local Holder exponent may vary from sample path to sample path in the case of a stochastic process, it is not a robust statistical tool for characterizing signal roughness. The notion of the *singularity spectrum* of a signal was introduced to give a less detailed but more stable characterization of the local smoothness structure of a function in the statistical sense.

Definition: *Let f: $R \rightarrow R$ be a real-valued function, and for each $\alpha > 0$ define the set of points at which f has a local Holder exponent h:*

$$\Omega(\alpha) = \{t, h_f(t) = \alpha\}$$

*The **singularity spectrum** of f is the function D:* $R^+ \rightarrow R$*, which associates to each* $\alpha > 0$ *the Hausdorff-Besicovich dimension of* $\Omega(\alpha)$*:*

$$D(\alpha) = dim_{HB}\Omega(\alpha)$$

2.3.2 Singularity Spectrum of a Stochastic Process

Using the above definition one may associate to each sample path $X_t(\omega) o$ a stochastic process X_t singularity spectrum $d_\omega(\alpha)$. If d_ω is "strongly dependent" on ω, then the empirical estimation of the singularity spectrum is not likely to give much information about the properties of the process X_t.

Fortunately, this turns out not to be the case: it has been shown that, for large classes of stochastic processes, the singularity spectrum is the same for almost all sample paths. Results from S. Jaffard [7] show that a large class of Levy processes verifies this property.

These results show the statistical robustness of the singularity spectrum as a nonparametric tool for distinguishing classes of stochastic processes. For example, it can be used as an explanatory statistical tool for determining the class of stochastic models that is likely to reproduce well the regularity properties of a given sample empirical path. But first one most know a method to estimate the singularity spectrum empirically.

2.3.3 Multifractal formalism

As defined above the singularity spectrum of a function does not appear to be of any practical use, since its definition involves first the $\Delta t \rightarrow 0$ limit for determining the local Holder exponents and second the determination of the Hausdorff dimension of the sets $\Omega(\alpha)$, as remarked by Halsey *et al.* [8], may be intertwined fractal sets with complex structures and impossible to separate on a point-by-point basis.

The work of Halsey *et al.* stimulated interest in the area of singularity spectra and a new multifractal formalism [8, 9, 10, 11] was subsequently defined and developed using the wavelet transform of Muzy *et al.* [12].

Three methods of calculation were developed: structure function method, wavelet partition function method, and wavelet transform modulus maxima. A detailed mathematical account of all three methods is given in [10], and their validity for a wide class of *self-similar* functions was proven by Jaffard [11].

2.3.4 Singularity Spectra of Asset Price Series

A first surprising result is that the shape of the singularity spectrum does not depend on the asset considered: all series exhibit the same "inverted parabola" shape observed by Fisher, Calvert, and Mandelbrot [13]. The spectra have support from 0.3 to 0.9 (with some variations depending on the data set) with a maximum centered between 0.55 – 0.60. Note that the 0.55 – 0.60 is the range of values of the Hurst exponent reported in many studies of financial time series using R/S or similar techniques, which is not surprising since the maximum *D(h)* represents the "almost everywhere" Holder exponent that is the one detected by "global" estimators such as R/S (methods for computing Hurst are defined in the Appendix). The Hurst exponent then is a global measure of risk, defined as the smoothness or unsmoothness an asset exhibits.

As in [13] we have supplemented our studies of the global estimators by applying the same techniques to Monte Carlo simulations of various stochastic models in order to check whether the peculiar shape of the spectra obtained is due to artifacts or small sample size or discretization. Using both daily and weekly log returns on a randomly selected set of 200 mutual funds from a population of 3,579, our results seem to rule out such possibilities. In addition, on our set of 200 funds we destroyed all the time dependencies that might exist in the data by shuffling the time series of each price return 19 times, thereby creating 19 new time series that contained statistically independent returns. The same global estimator routines were then run against these data (19 x 200), and in each case the Hurst exponent was not statistically different from 0.50, as expected. This was another confirmation that time-dependent volatility was the cause of the scaling behavior captured by Hurst.

3. Equity Curves and Risk-Adjusted Return

An appropriate performance measure is the most crucial determinant in judging the performance of investment strategies. Whether one is running a desk, investing on one's own or managing a pension fund, return on capital and the risk incurred to reach that return on capital must be measured together.

Ideally, a good performance measure should show high performance when the return on capital is high, when the equity/return curve increases linearly over time, and when loss periods (if any) are not clustered.

Unfortunately, common measurement tools such as the Sharpe Ratio (*SR*), Tracking Error (*TE*), and the Information Ratio (*IR*) do not entirely satisfy these requirements.

First, *SR* and *IR* put the variance of the return in the denominator, which makes the ratio numerically unstable at extremely large values when the variance of the return is close to zero. Second, *SR*, *TE,* and *IR* are unable to consider the clustering of profits and losses. An even mixture of profits and losses is normally preferred to clusters of losses and clusters of profits, provided the total set of profit and loss trades is the same in both cases. Third, all three measures treat the variability of profitable returns the same way as the variability of losses. Most investors, portfolio managers, and traders are more concerned about the variability of losses than they are about the variability of profitable returns.

3.1 Equity Curves

The equity curve is the cumulative value of all closed trades. Therefore, the equity curve on a monthly basis is the sum of all the closed trades over the trading horizon, while the equity curve on a yearly basis is the sum of all closed trades on a yearly basis.

To evaluate any fund's performance, the equity curve must be taken into account. The reasons for this are several. Perfect profit, which is defined as buying every valley and selling every peak that occurs in the price movement:

$$PP = \Sigma abs[(NAV_t - NAV_{t-1})/ NAV_{t-1}]$$

where NAV_t and NAV_{t-1} are the net asset values (NAVs) of the fund (with distributions reinvested) at t and $t-1$, is one of the tools used on trading desks to evaluate trader performance. As noted above, mathematically perfect profit is the sum of the absolute price differences and, obviously, impossible to obtain (hence the name *perfect* profit).

A desk manager could use the perfect profit and the equity curve of any trader and compute the correlation coefficient of the two. A value near plus 1 would indicate that as perfect profit is increasing, so is the trader's equity curve. A value of minus 1 would indicate that as perfect profit is increasing, the equity curve is decreasing. Desk managers consider this tool of value because perfect profit is a cumulative measure and will therefore be growing throughout the trading period. A good trader, and for that matter, a good pension fund manager or a good portfolio manager, will show a steadily rising equity curve. If the growth in the market becomes quiet, growth in perfect profit will tend to increase at a slower rate. The best trader will also share a similar flattening or slow growth instead of a dip in the equity curve over the period. This measure then, unlike common standalone measures such as *net profit and loss*, *rate of return,* and *maximum drawdown* will favor traders that steadily profit at the same pace as the perfect profit growth and do not lose much when perfect profit slows. As a sole guide of performance it is very valuable, and it is also a good candidate for a desk or firm with a low threshold for risk.

In the area of mutual fund or pension fund manager performance, it is well nigh impossible to compute perfect profit, since most funds do not release their securities holdings on a monthly basis (and when they do, it is with some delay), and also because taking a short position in the market is either severely limited or banned entirely. A different tool that uses the equity curve but can take these data and policy limitations into account is needed.

3.2 Effective Return

Effective Return *(*ER*)* was introduced in a paper by Dacorogna *et al.* [14]. Their performance measure is based on some assumptions that are different from those found in the literature. SR, as a conventional measure, stays approximately constant if the leverage of an investment is changed. Therefore, it cannot be used as a criterion to decide on the choice of leverage.

Real investors, however, care about the optimal choice of leverage because they do not have an infinite tolerance for losses.

Finding the maximum of ER for a mutual fund in a set of investment strategies is equivalent to portfolio optimization, where the allocation size (leverage) of different funds is determined. There is a strong relationship between the ER measure and classical portfolio theory. The main goal of portfolio optimization is to find the maximum of the return $r(t,T)$ for a given variance σ, or, equivalently, the maximum of the joint target function:

$$Max = E(r(t,T)) - \lambda\sigma^2$$

where λ is the Lagrange multiplier. ER, in its constant risk aversion form, is:

$$ER = E(r(t,T)) - (\gamma\sigma^2/2\Delta t)$$

The risk aversion parameter γ plays a role analogous to the Lagrange multiplier λ and the second term, $-\lambda\sigma^2$, in the Markowitz model, performs the same task as the corresponding term in ER and both have a natural interpretation: it is the risk premium associated with the investment.

In general, as shown above in Section 2, returns cannot be expected to be serially independent. Loss returns may be clustered to form drawdowns. The clustering of losses varies, i.e., it may be stronger for certain markets and/or investment strategies versus others. The ER measure has been designed with special attention to drawdowns, since these are the worst events for investors. The “badness” of a drawdown is mainly determined by the size of the total loss. Local details of the equity curve and the duration of the drawdown are viewed as less important. Multiple holding periods are examined because a constant/single holding period, i.e., choosing just a single monthly, daily, or yearly holding period, may miss drawdown periods when the interval size is too small for the full clustering of losses or too large, thus diluting the drawdown with surrounding profitable periods. The multi-horizon feature of ER ensures the worse drawdown periods cannot be missed, whatever their duration.

ER, in both Dacorogna *et al.* and the tests below, has been shown to be a more stringent performance measure relative to SR, net profit and loss, and maximum drawdown because it utilizes more points of the equity curve. The complete development of the measure is presented in [14], while in the Appendix of this paper the base methodology or single-horizon effective return methodology is developed.

4. Methodological Tests

In this section the results of two methodological tests will be presented: first, a test using the Hurst Exponent (H) and standard deviation as a way of constructing mean-variance portfolios, and second, a test of H versus SR in determining what funds to buy. Data sources and filters used to "scrub" the data will also be described.

4.1 Data

The source of all the data used in this section is Lipper, a fund analysis service provider. Lipper tracks both U.S. and non-U.S. funds on a daily, weekly, and monthly basis. Its database is quite extensive; it includes not just return data but expenses, manager tenure, and type of fund as well as many other measures.

The Lipper data used here are the universe of open-end equity funds that have at least three years of daily NAV data. Daily log returns $r(t,T)$ are computed using NAVs adjusted for the reinvestment of distributions. Where a fund has multiple classes of shares, the share class with the oldest FPO (first public offering) date is used. Also, institutional classes of shares are excluded so the results presented are for retail funds only.

4.2 Mean-Variance Test

The first test examines the use of H as the quantity to be minimized in a mean-variance portfolio. Since the higher the H value the lower the risk, H is inverted as an input to the allocation methodology to preserve its properties.

The measure-variance allocation methodology was executed in Excel and then checked against a similar routine available as an applet on Bill Sharpe's Web site. The results of the optimization routine were found to be the same, so no optimization methodological breaks were found.

The portfolio to be optimized is the whole market, i.e., funds that represents the complete stock, bond, and real estate markets. Both Vanguard and Fidelity offer funds that track the indices representing the complete markets: the Wilshire 5000, the Lehman Brothers Aggregate Bond Index, and the Morgan Stanley REIT Index. Vanguard funds were chosen over Fidelity funds, since they had the most complete history over the test horizon. The NASDAQ tickers for the funds are: VGSIX for the REIT series, VTSMX for the Wilshire 5000 series, and VBMFX for the Lehman Brothers Aggregate Bond series. The Vanguard REIT fund started in May 1996, so data runs prior to that date were supplemented with data from the Morgan Stanley index itself.

The parameters of the mean-variance input for the funds were average daily log returns over the prior three years, the linear correlation coefficient of each fund versus the other funds, and either H based on three years of daily data or the sample standard deviation based on three years of daily data.

Cash was assumed to be a minimum 5% of the portfolio and was represented by the rate on 3 month U.S. T-Bills, the most common choice for this variable. The risk aversion parameter was varied between 8.0 and 12.5 to test different risk aversion levels. The 12.5 parameter was chosen as the maximum level, since levels beyond that, in most cases, produced identical portfolios.

Portfolios were formed on a monthly basis from 1996 through 2000. Each portfolio was evaluated after 12 months via its SR levels as well as its maximum drawdown. The results are given in Table 1 with monthly results averaged across risk aversion levels and net profit and loss annualized:

Table 1

PASSIVE INVESTING - BUYING THE DOMESTIC MARKET
(12 month holding periods)

1996 Results	Net profit/loss	Max. Drawdown	Sharpe ratio
Minimizing Stdev	21.73	13.67	2
Minimizing H	33.2	9.12	2.18
1997 Results	**Net profit/loss**	**Max. Drawdown**	**Sharpe ratio**
Minimizing Stdev	23.34	13.78	1.58
Minimizing H	28.86	12.65	1.92
1998 Results	**Net profit/loss**	**Max. Drawdown**	**Sharpe ratio**
Minimizing Stdev	14.3	21.27	0.77
Minimizing H	9.49	7.46	1.31
1999 Results	**Net profit/loss**	**Max. Drawdown**	**Sharpe ratio**
Minimizing Stdev	6.94	8.54	0.79
Minimizing H	4.57	0.15	2.92
2000 Results	**Net profit/loss**	**Max. Drawdown**	**Sharpe ratio**
Minimizing Stdev	-5.35	19.68	-0.23
Minimizing H	0	0.53	1.78

As can be seen, in each year the H minimized portfolios outperformed the standard deviation portfolios on a risk-adjusted basis and they also had the smaller maximum drawdowns. Though only summary data are presented here, in most months the H minimized portfolio bested the standard deviation portfolio on the same two measures.

Though the most obvious explanation for this would be H's superior ability to detect the clustering of losses (tests were run over the same period, choosing portfolios based on ER with similar results), another explanation is possible as well.

In recent work by Johansen *et al.* [15], several U.S. and ex-U.S. equity indices were found to follow power-law/scaling behavior with superimposed log periodic oscillations. The power-law/scaling behavior is adequately captured by H, but log periodic oscillations would seem to be beyond the ken of H. However, judging from the limited tests done here, it appears that H, via its almost-everywhere smoothness computation, began to detect the growing volatility of the stock index by late 1999 and began to make the shift out of stocks and into REITs and bonds by late 1999. By mid- to late-2000, the bull market in bonds was detected by H, and a second shift occurred, with cash and bonds getting the largest allocations. Sample portfolios are shown below to illustrate this (the first line in each quarter is the standard deviation portfolio, while the second line in each quarter is the H portfolio).

Table 2

What was bought:

Oct-99

Wilshire 5000	REITs	Lehman Agg	Tbills
46%	7%	42%	5%
38%	57%	0	5%

Dec-99

Wilshire 5000	REITs	Lehman Agg	Tbills
66%	29%	0	5%
15%	54%	27%	5%

Mar-00

Wilshire 5000	REITs	Lehman Agg	Tbills
82%	13%	0	5%
5%	42%	25%	28%

Jun-00

Wilshire 5000	REITs	Lehman Agg	Tbills
46%	0	40%	14%
0	15%	35%	50%

Sep-00

Wilshire 5000	REITs	Lehman Agg	Tbills
85%	15%	0	0
0	3%	25%	85%

This is not to say the log periodic oscillations described by Johansen *et al.* are completely captured by H, but H does appear to have some "early warning" signal abilities if this limited test is correct.

4.3 Hurst, Effective Return, and the Sharpe Ratio

Our second test was to see if a commonly used technique to evaluate funds, SR, was superior to, the same as, or worse than using H and ER together.

Again, Lipper data were used to compute log returns, and this time excess returns, i.e., the fund's daily return versus three-month T-Bills (as measured by the Merrill Lynch index), were used.

Again, the universe of open-end equity funds with at least three years of daily NAVs was used with the filters mentioned above, e.g., oldest FPO date for a multiple share-class fund, implemented.

As in the mean-variance tests, portfolios were formed on a monthly basis from 1998 through 2000, and their 12-month performance was evaluated.

For the H/ER funds, the minimum H value allowable was 0.70, and the minimum ER allowable was 1%, i.e., with an upwardly sloping equity curve/P&L.

ER was computed on three years of daily excess returns (as was H), with ER horizons chosen based on 2^n, i.e., 2 days, 4 days, 8 days, …, 512 days. The weightings were centered on $n = 8$ or 256 days, approximately 1 year.

The Sharpe Ratio (SR) was computed via:

$$SR = E(X - B)/\sigma(X - B)$$

where X is equal to the daily log returns of the fund, and B is equal to the daily log returns of the three-month T-bill.

A minimum of three funds was chosen in each month, with the maximum number of funds set equal to ten.

Results of the tests were averaged across months with net profit and loss annualized.

Table 3

	Net profit/loss	Max. drawdown	Sharpe ratio
1998			
H/ER	12.21	11.02	1.87
Sharpe	11.09	17.73	1.30
1999			
H/ER	1.50	11.37	0.21
Sharpe	-1.50	12.03	-0.30
2000			
H/ER	-10.21	13.06	0.35
Sharpe	-20.02	16.08	-1.69

As can be seen, the results of these tests again show the efficacy of H and ER versus a traditional tool such as the SR. Though the H/ER outperforms SR in each of the three years, it is the last year, 2000, that shows H/ER's robustness in countering downdrafts. Note how the average maximum drawdown is substantially better for the H/ER funds versus the SR in each of the years.

5. Conclusions

As the above tests show H, either by itself or coupled with ER, is an effective means of choosing funds on an *unconditional* basis. It can also be said that H is probably more effective in terms of protecting an investor from downdrafts–the clustering of losses–than a traditional measure like SR. H and the ER methodology used here are conservative tools. What we mean by this is that for those investors who have a low threshold for risk, i.e., a moderate level of risk aversion, H and/or ER are good choices for screening funds.

We also think our tests agree with the statement by Mandelbrot that H is the *intrinsic measure of volatility* when volatility is defined as the variability or *(un)smoothness* of the sample path.

Given that on a daily basis markets are clearly not Gaussian or even random independent and identically distributed (i.i.d.), the methodologies outlined in this paper and elsewhere need to be used to evaluate and manage market risk and should also be used as the starting points for elaborating on stochastic models of asset prices.

Our continuing work on time intervals other than daily, i.e., weekly and monthly holding periods, have shown that for mutual funds the assumption of a normal distribution is not justified in as many as 50% of all cases. New tools are being developed to work with these emerging facts, and new economic theory, especially a revision to the EMH, is called for. To paraphrase Andrew Lo, EMH does not necessarily imply random and i.i.d. Modifications are called for. Interested readers are referred to Olsen *et al.* [18] for the direction that explanation might take.

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APPENDIX

A. Computing Hurst: *from Taqqu et al. [16]*

- For time series data the *rescaled range statistic (R/S)* is computed in the following manner: for a time series of length N, fit an $AR(1)$ process to the data, take the N-1 or M residuals, and subdivide the residual series into K blocks, each of size M/K. Then for each lag n, compute $R(k_i,n)/S(k_i,n)$, starting at points $k_i = iM/K + 1,\ I = 1,2,\ldots,$ such that $k_i + n \leq M$. $R(k_i,n)/S(k_i,n)$ is equal to the partial sum of the M residuals: $Y(n) = \Sigma^n{}_{i=1}M_i$ and sample variance: $S^2(n) = (1/n)\ \Sigma^n{}_{i=1}M_i^2 - (1/n)^2Y(n)^2$. $R(k_i,n)/S(k_i,n)$ is computed via:

$$R(k_i,n)/S(k_i,n) = 1/S(n)[max(Y(t)\text{-}t/nY(n))\text{-}min(Y(t)\text{-}t/nY(n))]$$

  For values of n smaller than M/K, one gets K different estimates of $R(n)/S(n)$. For values of n approaching M, one gets fewer values (as few as 1 when $n \geq M\text{-}M/K)$.

  Choosing logarithmically spaced values of n, plot log $R(k_i,n)/S(k_i,n)$ versus log n and get, for each n, several points on the plot. H can be estimated by fitting a line to the points of the log-log plot. Since any short-range dependence typically results in a transient zone at the low end of the plot, set a cut-off point, and do not use the low-end values of the plot for computing H. Often, the high-end values of the plot are also not used because there are too few points with which to make reliable estimates. The values of n then lie between the lower and higher cut-off points, and it is these points that are used to estimate H. A routine such as Least-Trimmed Squares Regression (LTS) has been used by the author to fit the data successfully.

- In the *periodogram method*, one first calculates:

$$I(\lambda) = 1/2\pi M\ /\Sigma X_j e^{ij\lambda}\ /^2$$

where λ is a frequency, N is the number of terms in the series, and X_j is the residual data as computed above. Because $I(\lambda)$ is an estimator of the spectral density, a series with long-range dependence should have a periodogram that is proportional to $/\lambda/^{1\text{-}2H}$ close to the origin. Therefore, a regression of the log of the periodogram versus the log λ should give the coefficient $1\text{-}2H$. This provides an approximation of H. In practice the author has used only the lowest 10% of the roughly $N/2 = 378$ frequencies for the regressions calculated in Section 4 above. This is because the above proportionality holds only for λ close to the origin.

The Hurst computed in Section 4 was the average H based on the R/S and periodogram values using 756 or three years of data points.

The Hurst tests discussed in Section 2.1.5 used eight years or 2,016 data points and were also the average Hurst as mentioned above.

B. Computing Effective Return *from Dacorogna et al.[14]*

- For effective return (ER) we shall assume that the investor has a stronger risk aversion to the clustering of losses, as was found by Benartzi and Thaler [17]. Thus the algorithm has two levels of risk aversion: a low one $\xi+$ for positive profit intervals (ΔR) and a high one $\xi-$ for negative ΔR (drawdowns):

$$\xi = \xi+ \text{ for } \Delta R \geq 0 \text{ and } \xi- \text{ for } \Delta R < 0 \text{ where } \xi+ < \xi-$$

The utility function is obtained by inserting the above equation into the definition of ΔR and integrating twice over ΔR:

$$U = U(\Delta R) = -e^{-\xi+\Delta R}/\xi+ \text{ for } \Delta R \geq 0$$

or

$$U = U(\Delta R) = (1/\xi-) - (1/\xi+) - (-e^{-\xi-\Delta R}/\xi-) \text{ for } \Delta R < 0$$

The return is obtained by inverting the utility function so that:

$$\Delta R = \Delta R(u) = -\log(-\xi+u)/\xi+ \text{ for } u \geq -1/\xi+$$

or

$$\Delta R = \Delta R(u) = -\log(1 - \xi-/\xi+ - \xi-u)/\xi- \text{ for } u < -1/\xi-$$

This is the complete development of the single horizon ER measure. The expansion of ER to a multiple horizon measure will not be given here. It is fully developed in [14], pages 10-11, and is based on the derivation of ER used here.

- END -